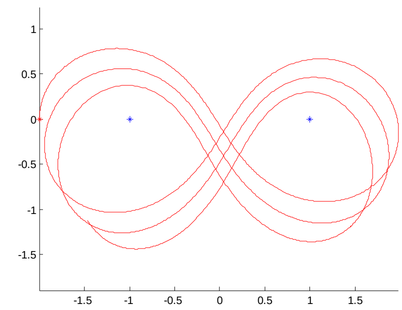
SM233 – Project 2– The Pseudo Three Body Problem
Project – Due – 31 March 2008



We shall consider a simplified version of the 3 body problem with two stationary bodies of mass m_1 and m_2 located at (x_1, y_1) and (x_2, y_2) respectively. A third body will be in motion traveling along a trajectory (x, y). An initial velocity (v_{x_0}, v_{y_0}) and position (x_0, y_0) will be given.

Additionally you must select a time of flight t and a time increment Δt. If we assume that the gravitational constant $G=1$, then equations of motion for this system are:

$$a_x = \frac{dv_x}{dt} = -m_1 \frac{x - x_1}{\left((x-x_1)^2 + (y-y_1)^2\right)^{3/2}} - m_2 \frac{x - x_2}{\left((x-x_2)^2 + (y-y_2)^2\right)^{3/2}}$$

$$a_y = \frac{dv_y}{dt} = -m_1 \frac{y - y_1}{\left((x-x_1)^2 + (y-y_1)^2\right)^{3/2}} - m_2 \frac{y - y_2}{\left((x-x_2)^2 + (y-y_2)^2\right)^{3/2}}$$

Using the improved Euler's method, we cam approximate and plot the trajectory of the moving mass. The following series of steps are suggested to help you write your program.

- Increment the position (i.e. $x_P(t+\Delta t) \approx x(t) + v_x \Delta t$, repeat for y)
- Increment the velocity (i.e. $v_x(t+\Delta t) \approx v_x(t) + a_x(x,y)\Delta t$, repeat for v_y)
- Average the velocities at the beginning and end of the increment (i.e. $v_{\bar{x}} = (v_x(t) + v_x(t+\Delta t))/2, \ldots$)
- Find the corrected position based on the average velocity (i.e. $x_C(t+\Delta t) \approx x(t) + v_{\bar{x}} \Delta t, \ldots$)
- Note that (x_P, y_P) are intermediate predictor values that are not part of the trajectory.
- (x_C, y_C) is the position of the moving object at the end of the increment.

Problems: Possible Gravitational Configurations in an Infinite Universe:

1. **The Gravitational Yo-Yo**. Place two object with $m=1$ at the points (-1, 0) and (1, 0). Place a third object with no initial velocity at the point (0, 1). Let the time of flight $t=40$ and the time increment $\Delta t = .02$. Plot the resulting trajectory. Now change the mass of the left object to 1.1 Plot the resulting trajectory. Can you explain what is happening?

2. **The Dance of the Figure Eight:** Repeat the initial configuration above, but let the initial point of the moving object be (-2,0) and the initial velocity be (0,.64635). Plot the resulting orbit. What happens to the trajectory if the initial velocity is (0,.62)? What can you conclude about the stability of special orbits?

3. **Include your code with your project.**