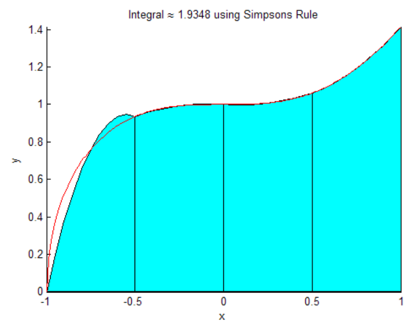
SM233 – Project D – Simpson's Rule

(Due Monday 2/25)



Background: Simpsons rule is used to numerically calculate the value of definite integrals that can not be calculated analytically. In the midpoint rule, we used function values from 2 increments to construct a rectangle. We than found the "area" of each rectangle and summed them to obtain an estimate of the integral. Simpson's rule constructs a parabola using points from three increments and than calculates the "area" under each parabola. Simpson's Rule is given by the formula:

$$\int_a^b f(x)dx \approx \frac{\Delta x}{3}\sum_{i=1}^{n}\left(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right)$$

The term "area" is used loosely here. If the function goes below the *x*-axis, the value of the "area" is negative.

Assignment: Write a program with inputs *f(x)*, *a*, *b*, *n* , where *n* is the number of points used to form the *x* vector. Outputs should include the approximation of the integral $\int_a^b f(x)dx$, a plot of the curve *f(x)*, and the parabola formed at each set of increments Note that it takes two increments or three points to plot each parabola. The title of each plot should report the value of the integral and the number of intervals used. Note that Simpson's Rule requires an even number of intervals (why?), and therefore your program should give you a warning if you attempt to run it for an odd number of intervals. Note that an odd number of intervals requires an even number of points … i.e. n==even (Why?). You can end a program with the command *return.*

Hint: Use elements from your curve fitting project to construct the parabolas. Use the fill command to draw the parabola (i.e. *fill([x_n, **X**, x_{n+2}, x_n], ([0, **Y**, 0,0]), 'color')*, where ***X*** and ***Y*** are vectors containing the points that make up the parabola).

Questions:

1. Approximate $\int_0^1 \sqrt{1+x^3}dx$. Use n=9 (i.e. 8 intervals which produces 4 parabolas).
2. Approximate $\int_0^{pi} \sin(x^2)\, dx$. Use n=5.
3. Approximate $\int_0^{pi} \sin(x^2)\, dx$. Use n=21.